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THE EFFECT OF TRANSFER PROCESSES ON THE STABILITY OF A PLANE FLAME FRONT

A. G. Istratov and V. B. Librovich

Derivation of a revised approximate solution, for large but finite Reynolds numbers, to prove the theory of Landau concerning absolute instability of a plane laminar flame front. The solution takes into account the effects of viscosity, diffusion, heat conduction, and variations in the chemical reaction rate on flame front stability. Corrections to order $1/N_R$ of Landau's solution are determined by the Germain and Guiraud method of studying transfer processes in weakly bent shock waves.

The absolute instability of a plane laminar flame front, proved by L. D. Landau [1, 2], is derived under the assumption that the Reynolds number along the length of the exciting wave is infintely large. This circumstance allowed L. D. Landau, during investigation of the instability, to examine the equations of an ideal fluid, while considering the flame front as an infinitesimally thin hydrodynamic discontinuity which is propagated with a known constant normal flame velocity with respect to the gas. /451*

This work is devoted to the determination of the next approximation to the solution of L. D. Landau, inverse by a small magnitude to the Reynolds number. In this approximation (the Reynolds number is large but not infinite) it is necessary to consider the influence on the stability of the flame of the transfer processes (viscosity, diffusion, heat transfer) which change the field of gas flow regardless of flame and themselves determine the constitution of the flame. Besides this, it is necessary to take into account the change in speed of the chemical reaction in the flame, which influences its speed of propagation with respect to the burning mixture.

The results, arrived at for a plain thermal mechanism of propagation of the flame, allow the determination of the critical length of the perturbing wave and the corresponding critical Reynolds number, for which the laminar flame is stable. The critical Reynolds number depends on the degree of thermal expansion of the gas during combustion, the Prandtl and Lewis numbers, and the normalized activation energy of the chemical reaction.

Let us note that works covering investigation of the effects of stabilizing factors on the stability of laminar flames [3-9] err through essential inadequacies. In the series of works [3, 4, 6] the authors manifested a lack of understanding of the asymptotic character of the theory of instability of L. D. Landau and because of this allowed discontinuities in calculations of corrections for perturbation of flames with finite Reynolds numbers. This criticism

*/Numbers in the margin indicate pagination of the original foreign text.

particularly applies to the deduction of boundary conditions on the surface of the flame -- not in one work was the influence of the finite thickness of the flame front taken into account -- this was explained by the fact that, up to the present no mathematical method has been worked out that allows for systematically taking into account corrections to solutions having discontinuities in the initial approximation. Only recently the works of P. Germain and J. Guiraud [10-12] appeared on slightly distorted shock waves. These works proposed a method of successive computation of corrections of the first order in the boundary conditions at hydrodynamic discontinuities. This method, adapted to flame, is utilized in this work.

Besides this, in the available works, efforts were made to at least take into account separate stabilizing effects, ignoring others having the same order of magnitude (for instance, diffusion and heat transfer were taken into account, but viscosity was ignored [3, 6, 7], or vice versa [5]). Let us stress that, under a step-by-step approach to the problem, it is necessary to take into account every one of the following effects, all of the same order $\varepsilon = 1/N_R$ ($N_R = u_n/k\nu$ representing the Reynolds number; u_n, the velocity of propagation of the flame; k, the perturbing wave number; and ν, the viscosity): 1) the influence of the flame curvature on the diffusion processes, heat transfer and viscosity in the wave front; 2) the change in the speed of combustion (the speed of chemical reaction in the flame) in connection with temperature propagation; 3) the /452 effect of the finite thickness of the flame front; 4) the change in density of the products of combustion behind the flame front due to temperature perturbation; 5) the effects of viscosity on the motion of the gas beyond the flame front. In the present work, all these aforementioned effects are taken into consideration.

The system of notation adopted in the work: the upper indices ^{0} and ^{1} relate, respectively, to the zero-th and first approximation in ε. Lower index (-) relates to solutions in the burning mixture, while the lower index (+) relates to the solution in the products of combustion. During linearization, quantities describing the stationary unperturbed solution are represented by capital letters, while their related perturbations by lower case primes: for the unperturbed solution the velocity is U, density R, pressure P, normalized temperature Θ, concentration S; the perturbed quantities are u', ρ', p', θ', s'. Lower case unprimed letters relate to the total magnitude of the respective quantities ($\rho = R + \rho'$). The remaining notations are explained in the text.

§1. We will carry out the identification of corrections of the order of $1/N_R$ to the solution of L. D. Landau by following the method proposed by G. Germain and J. Guiraud for computing the transfer process for slightly distorted shock waves [10-12].

In conformance with this method, it is necessary to identify two regions of gas flow: a region of external flow (outside the flame) and a region within the flame (Figure 1, in which are drawn lines of gas flow during passage through the curved front of a laminar flame). In the region of external flow, the gradients are of small magnitude (if the magnitudes involved in the problem are normalized with respect to the length of the perturbing wave and the normal

velocity of flame perturbation, then these gradients are approximately unity), and therefore, in this region, the usual method of successive approximations can be applied. In other words, here all the magnitudes can be found in the form

$$f = f^0 + \varepsilon f^1 \tag{1.1}$$

where f^0 is the solution of the zero-th approximation (solution of L. D. Landau), while f^1 is the correction; f^1 is found from the corresponding equations through substitution in them of a solution in the form (1.1).

In the region inside the flame, the thickness of which is of the order $\varepsilon = 1/N_R$, insofar as it is determined similarly by transfer processes, the iteration method is not applicable -- the magnitude of the gradients in this region are large -- approximately $1/\varepsilon$.

Nevertheless, we can calculate the transfer process in the flame zone if we take into consideration the narrowness of the zone as compared to the sizes of the zones of the perturbed flow of gas (external zones). For this, it is necessary for solution in the external region to set up boundary conditions which would take into account the structure and thickness of the flame front, i.e., consider the influence of the internal zone by means of integration. In this, the boundary conditions on the flame surface in the general case will be written in the form

$$[f^0] + \varepsilon [f^1] = \varepsilon \Delta (f^0) \tag{1.2}$$

where $[f^0]$ is the magnitude of the discontinuity on the flame front, corresponding to the solution of the problem in the zero-th approximation, while $[f^1]$ and $\Delta(f^0)$ are the corrections connected to the transfer process and the finite thickness of the flame front.

Establishing the connection between the solutions in the region of cold gas and the region of the products of combustion, having the form (1.1), with the help of boundary conditions (1.2) and equating the coefficients between the first and second levels of ε, we can get two systems of homogeneous linear algebraic equations. The first system (zero-th approximation) corresponds to the solution of L. D. Landau and serves to determine the characteristic frequency in the theory of L. D. Landau and the relationship between the amplitudes of pressure perturbation and the velocities along both sides of the flame and the amplitude of the perturbed surface of the flame. The second system (first approximation) allows the determination of the amplitudes of the first approximation through the amplitude of the perturbed surface of the flame, and similarly the change in characteristic frequency of the problem.

It is evident that in the solution of the problem in the first approxi- /453
mation the change in the characteristic frequency has the form

$$\omega = \omega^0 [1 - \varepsilon\tau(\sigma, L, \alpha, z)] \qquad (z = E/2R_0T_b) \tag{1.3}$$

Here ω is the normalized characteristic frequency; ω^0 is the zero-th approximation of the characteristic frequency; τ is the correction of the first approximation; α is the density ratio of the hot to cold gas; σ is the Prandtl number; L is the ratio of thermal conductivity to the diffusion coefficient of the combustible (Lewis number); z is the normalized activation energy determining the dependance of the speed of chemical reaction in the flame on the temperature; E is the activation energy; and T_b is the temperature of the products of combustion. It is assumed that there is room for a thermal mechanism of flame propagation in which the reaction is limited by a single flaming component.

The problem, in this manner, resolves itself into the determination of the function $\tau(\sigma, L, \alpha, z)$.

Strictly speaking, the solution for frequency (1.3) allows development of only the tendency for change of natural frequency under perturbation with finite Reynolds numbers. In order to get a change in sign for frequency, it is necessary to extrapolate expression (1.3) to those perturbing wavelengths for which ε_τ (σ, L, α, z) are approximately 1 (then τ represents the critical Reynolds number).

Let us note that for carrying out the iteration in the external region and for development of boundary conditions on the flame front it is possible to utilize the same equations for preservation of mass, reacting material, energy and amount of motion. In his work, L. D. Landau used only the equations for preservation of mass and amount of motion. This is explained by the fact that in solving the problem of flame stability in the zero-th approximation, the equations for preservation of reacting material and energy reduce to the trivial equations of thermal transfer and reacting material together with gas flow; solution of these equations, written for perturbed concentrations of reacting material and temperatures, give identically zeros.

Because of this, these equations can be omitted without loss. In the boundary conditions on the flame, only a negligible effect from the equation for preservation of reacting material remains -- conditions of constant normal flame velocity (as a matter of fact, this condition of constant velocity is a requirement of a reacting material in a chemical reaction). During solution of the problem of flame stability, in the next approximation it is necessary to take into account transfer of energy, material and amount of motion due to convection, diffusion and viscosity. Then the equation for preservation of energy and reacting material will not lead to trivial solutions and they must therefore be considered.

§2. The basic equations for the problem are the equations for the preservation of mass, amount of motion, energy and reacting material

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\varepsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\varepsilon}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\varepsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{\varepsilon}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right).$$

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}=-\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\rho\frac{\partial\theta}{\partial t}+\rho u\frac{\partial\theta}{\partial x}+\rho v\frac{\partial\theta}{\partial y}=\frac{\varepsilon}{\sigma}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{w}{\varepsilon}\qquad\left(\theta=\frac{T-T_0}{T_b-T_0}\right)$$

$$\rho\frac{\partial s}{\partial t}+\rho u\frac{\partial s}{\partial x}+\rho v\frac{\partial s}{\partial y}=\frac{\varepsilon}{\sigma L}\left(\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}\right)-\frac{w}{\varepsilon}\qquad(2.1)$$

In the equations, the following designations are used: x and y represent /454 normalized space coordinates (see Figure 1) relative to the length of the perturbation wave, i.e., more exactly, to $k = 2\pi/\lambda$ (k being the wave number). t is the normalized time, determined as the ratio of the time to the characteristic time $1/u_n k$; u and v are the normalized components of velocity in units of normal velocity of flame propagation u_n; p is the normalized pressure (pressure, ratioed to the dynamic pressure head of fresh mixture); ρ is the normalized density (related to the density of cold gas); s is the relative concentration of reacting matter; θ is the normalized temperature; T_0 is the original temperature of the mixture; T_b is the temperature of the products of combustion during burning without motion; and ω is the normalized speed of chemical reaction. It is similarly assumed that the heat transfer, dynamic viscosity, specific heat and the products of the diffusion coefficient and density remain constant, regardless of temperature and concentration.

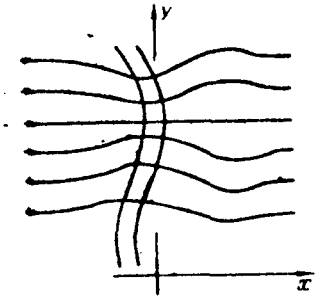

Figure 1.

As can be seen from the last equations (2.1), in the expression for the speed of chemical change the parameter ε appears. Why this parameter appears in the denominator becomes obvious from the following considerations. From the general statement of the dimensional theory it follows that the normal velocity of propagation of the flame is connected to the thermal transfer of the gas $\varkappa$ and the speed of the chemical reaction W(T, s) with relation to $u_n \sim (\varkappa W)^{1/2}$, or, in its turn, $W \sim u_n^2/\varkappa$. Normalizing this relation into the normal velocity of the flame and the propagation wave number, we come to the expression which is used in equations (2.1). This expression properly reflects the evident fact that during the approach of ε to zero the speed of heat transfer must turn into a delta function connected to an infinitesimally thin flame front.

Beside equations (2.1), it is essential to draw attention to the equation of state of the gas which, for the case of an incompressible gas (flame velocity small in comparison to speed of sound), reduces to a relation between the gas density and its temperature

$$\rho = 1 \Big/ \left(1 + \frac{1-\alpha}{\alpha}\theta\right) \tag{2.2}$$

Investigation of stability will be carried out by the method of small perturbations. We shall seek a solution of equations (2.1) in the form

$$f = F + f' = F(x) + [f^0(x) + \varepsilon f^1(x)] \exp(iy + \omega t) \tag{2.3}$$

Substituting the solution of form (2.3) in the equations, we will leave in them terms of the order ε. Thus, simplifying through overall multiplication by $\exp(iy + \omega t)$, we get

$$R\omega u^0 + \varepsilon R\omega u^1 + \frac{\partial u^0}{\partial x} + \varepsilon \frac{\partial u^1}{\partial x} = -\frac{\partial p^0}{\partial x} - \varepsilon \frac{\partial p^1}{\partial x} + \varepsilon\left(\frac{\partial^2 u^0}{\partial x^2} - u^0\right) + \frac{\varepsilon}{3}\frac{\partial}{\partial x}\left(\frac{\partial u^0}{\partial x} + iv^0\right)$$

$$R\omega v^0 + \varepsilon R\omega v^1 + \frac{\partial v^0}{\partial x} + \varepsilon \frac{\partial v^1}{\partial x} = -ip^0 - i\varepsilon p^1 + \varepsilon\left(\frac{\partial^2 v^0}{\partial x^2} - v^0\right) + i\frac{\varepsilon}{3}\left(\frac{\partial u^0}{\partial x} + iv^0\right)$$

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$$\omega R(\theta^0 + \varepsilon\theta^1) + \frac{\partial}{\partial x}(\theta^0 + \varepsilon\theta^1) = \frac{\alpha}{1-\alpha}\left[\frac{\partial}{\partial x}(u^0 + \varepsilon u^1) + i(v^0 + \varepsilon v^1)\right]$$

$$R\omega\theta^0 + \varepsilon R\omega\theta^1 + \frac{\partial\theta^0}{\partial x} + \varepsilon\frac{\partial\theta^1}{\partial x} = \frac{\varepsilon}{\sigma}\left(\frac{\partial^2\theta^0}{\partial x^2} - \theta^0\right)$$

$$R\omega s^0 + \varepsilon R\omega s^1 + \frac{\partial s^0}{\partial x} + \varepsilon\frac{\partial s^1}{\partial x} = \frac{\varepsilon}{\sigma L}\left(\frac{\partial^2 s^0}{\partial x^2} - s^0\right) \tag{2.4}$$

In the equations of continuity, magnitude $\rho^0 + \varepsilon\rho^1$ is eliminated with the help of relation (2.2). In the last equations, terms in ω/ε are neglected -- outside the flame the speed of reaction equals zero.

In order to arrive at the solution for the zero-th approximation in ε (solution of L. D. Landau) from the resulting equations, it is necessary in all the equations to set $\varepsilon = 0$ and solve them using the boundary conditions

$$u_-^{0,1},\ v_-^{0,1},\ p_-^{0,1},\ \theta_-^{0,1},\ s_-^{0,1} \to 0 \quad \text{as } x \to -\infty \text{ (cold gas)} \tag{2.5}$$
$$u_+^{0,1},\ v_+^{0,1},\ p_+^{0,1},\ s_+^{0,1} \to 0,\ |\theta_+^{0,1}| < \infty \text{ as } x \to +\infty \text{ (products of combustion)}$$

For the cold gas region with this we get the expression ($x < 0$)

$$u_-^0 = A^0 e^x, \quad v_-^0 = iA^0 e^x, \quad p_-^0 = -A^0(\omega + 1)e^x \tag{2.6}$$
$$\theta_-^0 = s_-^0 = \rho_-^0 = 0 \qquad (A^0 = \text{integration constant})$$

To evaluate terms of order ε it is necessary to equate in equations (2.4) the coefficients in ε and to substitute in them the calculated expressions for the zero-th approximation. For the cold gas region this procedure leads to the following equations for u_-^1, v_-^1, p_-^1, θ_-^1, s_-^1

$$\omega u_-^1 + \frac{\partial u_-^1}{\partial x} = -\frac{\partial p_-^1}{\partial x}, \qquad \omega v_-^1 + \frac{\partial v_-^1}{\partial x} = -ip_-^1$$
$$\frac{1-\alpha}{\alpha}\left(\omega\theta_-^1 + \frac{\partial \theta_-^1}{\partial x}\right) = \frac{\partial u_-^1}{\partial x} + iv_-^1, \quad \omega\theta_-^1 + \frac{\partial \theta_-^1}{\partial x} = 0, \quad \omega s_-^1 + \frac{\partial s_-^1}{\partial x} = 0 \tag{2.7}$$

Solution of these equations with boundary conditions (2.5) gives identically the self-same equations as does the solution in the zero-th approximation (2.6); the difference is centered only in a new constant of integration A^1.

In this manner, even in the first approximation in ε in the cold gas there is no perturbation in temperature, concentration and density. The transfer of reactive matter and of thermal energy is brought about exclusively through convection; diffusion and heat transfer processes become significant only in the second approximation.

For gas flow behind the flame front (in the region of the hot products of combustion) ($x > 0$) solution of equations (2.4) leads to the following results

in the zero-th approximation

$$u_+^0 = B^0 e^{-x} + C^0 e^{-\alpha\omega x}, \qquad v_+^0 = -iB^0 e^{-x} - i\alpha\omega C^0 e^{-\alpha\omega x}$$
$$p_+^0 = B^0(\alpha\omega - 1)e^{-x}, \quad \theta_+^0 = H^0 e^{-\alpha\omega x}, \quad \rho_+^0 = -\alpha(1-\alpha)\theta_+^0$$
$$s_+^0 = 0 \qquad (B^0,\ C^0 \text{ and } H^0 = \text{integration constants}) \tag{2.8}$$

Let us turn our attention to the following circumstances. The flow of gas behind the flame front is turbulent (in contrast to the flow ahead of the flame front), vortices develop on the flame and behind the flame are carried by the gas flow (in the expressions for the components of velocity, terms with constant C^0 correspond to turbulent gas flow). Besides this, there are possible temperature perturbations behind the flame front and, associated with temperature, perturbations in density which arise from convective transfer originating on the flame front from temperature perturbations. /456

In the following, the first approximation, the equations for hot gas have the following form

$$\alpha\omega u_+^1 + \frac{\partial u_+^1}{\partial x} = -\frac{\partial p_+^1}{\partial x} + [(\alpha\omega)^2 - 1]C^0 e^{-\alpha\omega x}$$
$$\alpha\omega v_+^1 + \frac{\partial v_+^1}{\partial x} = -ip_+^1 + i\alpha\omega[1 - (\alpha\omega)^2]C^0 e^{-\alpha\omega x}$$

$$\frac{1-\alpha}{\alpha}\left(\alpha\omega\theta_+^1+\frac{\partial\theta_+^1}{\partial x}\right)=\frac{\partial u_+^1}{\partial x}+iv_+^1,\quad \alpha\omega\theta_+^1+\frac{\partial\theta_+^1}{\partial x}=\frac{1}{\sigma}\left[(\alpha\omega)^2-1\right]H^0e^{-\alpha\omega x}$$

$$\rho_+^1=-\alpha(1-\alpha)\theta_+^1,\qquad s_+^1=0 \tag{2.9}$$

the solutions of which are

$$u_+^1=B^1e^{-x}+C^1e^{-\alpha\omega x}+\left[(\alpha\omega)^2-1\right]C^0xe^{-\alpha\omega x}$$
$$v_+^1=-iB^1e^{-x}-i\alpha\omega C^1e^{-\alpha\omega x}-i\left[1-(\alpha\omega)^2\right](1-\alpha\omega x)\,C^0e^{-\alpha\omega x}$$
$$p_+^1=B^1(\alpha\omega-1)e^{-x},\quad \rho_+^1=-\alpha(1-\alpha)\theta_+^1$$
$$\theta_+^1=H^1e^{-\alpha\omega x}+\frac{1}{\sigma}\left[(\alpha\omega)^2-1\right]H^0xe^{-\alpha\omega x} \tag{2.10}$$

Here B', C' and H' are the new constants of integration. In these solutions terms appear that characterize the transfer of thermal energy by heat conduction, and similarly the influence of viscosity behind the flame front.

Finally, we will designate the form of the curved flame front as

$$\zeta=D\exp(iy+\omega t) \tag{2.11}$$

The determination of the factors entering into the identification of the constant of integration must be performed with the help of the boundary conditions on the flame surface which establish the connection between the solutions in the regions of cold and hot gases. These boundary conditions should also be found with an accuracy within the first approximation of the magnitude ε.

§3. On the surface of the flame the conditions of conservation of tangential and normal components of impulse, mass, thermal energy and concentration of reactive matter must be observed (the last two conditions, while taking into account the isolation of heat in the flame and its requirement for reactive matter). Insofar as these conditions, expressing the laws of conservation, are contained in differential form in equations (2.1), derivation of boundary conditions can also utilize the indicated equations.

Let us illustrate the derivation of boundary conditions in the example of the conditions of conservation of tangential components of impulse. For this, let us take the first equation (2.1), rewriting it in the divergent form

$$\frac{\partial\rho u}{\partial t}=\frac{\partial}{\partial x}\left[-p-\rho u^2+\varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[-\rho uv+\varepsilon\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right] \tag{3.1}$$

Let us integrate this equation along x from $\zeta-\delta$ to $\zeta+\delta$ (near the surface of the flame). During this, let us select $\delta \gg \varepsilon$, so that the whole flame /457

structure will be included in the limits of integration. Making use of the rules of differentiation of integrals along a parameter, we have

$$\frac{\partial}{\partial t}\int_{\zeta-\delta}^{\zeta+\delta}\rho u\,dx-[\rho u]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial\zeta}{\partial t}=\left[-p-\rho u^2+\varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}\right)\right]_{\zeta-\delta}^{\zeta+\delta}+$$
$$+\frac{\partial}{\partial y}\int_{\zeta-\delta}^{\zeta+\delta}\left[-\rho uv+\varepsilon\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]dx-\left[-\rho uv+\varepsilon\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial\zeta}{\partial y} \tag{3.2}$$

Here, the differences in the quantities contained in the brackets on both sides of the flame are identified by the symbol $[\ ]_{\zeta-\delta}^{\zeta+\delta}$.

Three types of quantities enter into the resulting relation (3.2). In the first, these are the customary differences of impulse current on the movable curved surface

$$[\rho u]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial\zeta}{\partial t},\qquad [-p-\rho u^2]_{\zeta-\delta}^{\zeta+\delta},\qquad [-\rho uv]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial\zeta}{\partial y} \tag{3.3}$$

In the second, these are corrections of the order ε, connected with the presence on the boundaries $\zeta - \delta$ and $\zeta + \delta$ of a viscous impulse current

$$\varepsilon\left[\frac{4}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}\right]_{\zeta-\delta}^{\zeta+\delta},\qquad -\varepsilon\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial\zeta}{\partial y} \tag{3.4}$$

These corrections should be identified, calculating the corresponding differentials from the solution for an ideal gas (formulas (2.6) and (2.8)).

In the third, there exist the terms

$$\frac{\partial}{\partial t}\int_{\zeta-\delta}^{\zeta+\delta}\rho u\,dx,\qquad \frac{\partial}{\partial y}\int_{\zeta-\delta}^{\zeta+\delta}\left[-\rho uv+\varepsilon\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]dx \tag{3.5}$$

Germain and Guiraud [10-12] first noted that these integrals also give corrections of the order ε, besides which, for their calculations, it is necessary to know the structure of the flame of the zero-th order in ε.

Let us clarify this, following works [10-12]. Any quantity f (density, velocity, temperature, etc.) discontinuous when $\varepsilon = 0$, with ε bounded can be presented in the form

$$f = \frac{1}{2}[f_+(x-\zeta, y, t, \varepsilon) + f_-(x-\zeta, y, t, \varepsilon)] + \frac{1}{2}[f_+(x-\zeta, y, t, \varepsilon) - f_-(x-\zeta, y, t, \varepsilon)]\,\Phi\left(\frac{x-\zeta}{\varepsilon}, y, t, \varepsilon\right) \tag{3.6}$$

in which $\Phi(\pm\infty, y, t, \varepsilon) = \pm 1$. Far from the discontinuity when $|x - \zeta| \gg \varepsilon$ the function f must coincide with the discontinuous solutions f_+ (when $x - \zeta > 0$) and f_- (when $x - \zeta < 0$) (during this, it is assumed that the approach of f to $f_\pm$ is exponential). Besides this, it describes the structure of the discontinuity when $|x - \zeta| \lesssim \varepsilon$, since the magnitude of ε is of the inherent scale that identifies the width of the discontinuity.

From this is seen that during integration of the quantity f along x outside the discontinuity only the values of f_+ and f_- will be significant while the result of the integration along the width of the discontinuity can be evaluated as εf^*, where f^* is any effective value of the given quantity inside the discontinuity, depending on its structure. For calculation of f^*, P. Germain /458 and J. Guiraud propose a method that is analogous to the method for development of the thickness of displacement in boundary layer theory. Specifically, let us present function f in the form $f_e + f - f_e$, where $f_e = f_+$ when $x - \zeta > 0$ and $f_e = f_-$ when $x - \zeta < 0$. Then

$$\int_{\zeta-\delta}^{\zeta+\delta} f\,dx = \int_{\zeta-\delta}^{\zeta+\delta} f_e\,dx + \int_{\zeta-\delta}^{\zeta+\delta} (f - f_e)\,dx \tag{3.7}$$

The first of these integrals always appears during development of boundary conditions on an infinitesimally thin discontinuity. In the second integral the integrand function is different from zero because of the uniqueness of the structure of the discontinuity, therefore for calculation of this integral we shall make the substitution $\xi = (x - \zeta)/\varepsilon$. We will get

$$\int_{\zeta-\delta}^{\zeta+\delta} (f - f_e)\,dx = \varepsilon \int_{-\delta/\varepsilon}^{\delta/\varepsilon} (f - f_e)\,d\xi \tag{3.8}$$

Since $f - f_e$ exponentially approaches zero outside the discontinuity, then, taking into consideration that $\delta \gg \varepsilon$, we can apportion the limits of integration from $-\infty$ to $+\infty$. In this way, we get the quantity

$$f^* = \int_{-\infty}^{\infty} (f - f_e)\,d\xi \tag{3.9}$$

The boundary conditions (3.2) give the relation between the quantities when $x = \zeta + \delta$ and $x = \zeta - \delta$. To get the conditions on the surface of the discontinuity, it is necessary to expand all the quantities in a series in δ and take the first terms of the expansion.

And so, the conditions for the conservation of the tangential component of flow of impulse current have the form

$$\varepsilon \frac{\partial}{\partial t}(\rho u)^* - [\rho u]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} = \left[-p - \rho u^2 + \varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)\right]_{\zeta-0}^{\zeta+0} -$$
$$- \varepsilon \frac{\partial}{\partial y}(\rho u v)^* - \left[-\rho u v + \varepsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} \qquad (3.10)$$

Analogously, we will get the conditions for continuity of the normal component of impulse current and the conditions for the conservation of mass current

$$\varepsilon \frac{\partial}{\partial t}(\rho v)^* - [\rho v]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} = \left[-\rho u v + \varepsilon\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]_{\zeta-0}^{\zeta+0} -$$
$$- \varepsilon \frac{\partial}{\partial y}(p + \rho v^2)^* - \left[-p - \rho v^2 + \varepsilon\left(\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x}\right)\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y}$$
$$\varepsilon \frac{\partial}{\partial t}\rho^* - [\rho]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} + [\rho u]_{\zeta-0}^{\zeta+0} + \varepsilon \frac{\partial}{\partial y}(\rho v)^* - [\rho v]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} = 0 \qquad (3.11)$$

In the same manner the equations of heat transfer and reactive matter can be dealt with. Into these boundary conditions will enter the integral from the heat liberation function

$$M = \int_{\zeta-\delta}^{\zeta+\delta} \frac{w}{e}\,dx = \int_{-\infty}^{+\infty} w\,d\xi \qquad (3.12)$$

Such an integral represents the amount of heat liberated in a unit of time on a unit flame surface. In the stationary state $M = 1$, for a nonstationary curved flame front the quantity M identifies the nonstationary speed of combustion. With consideration of this circumstance the conditions for conservation of thermal current and reactive matter will take the form /459

$$\varepsilon \frac{\partial}{\partial t}(\rho\theta)^* - [\rho\theta]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} =$$
$$= \left[-\rho u\theta + \frac{\varepsilon}{\sigma}\frac{\partial \theta}{\partial x}\right]_{\zeta-0}^{\zeta+0} - \varepsilon \frac{\partial}{\partial y}(\rho v\theta)^* - \left[-\rho v\theta + \frac{\varepsilon}{\sigma}\frac{\partial \theta}{\partial y}\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} + M$$

$$\varepsilon \frac{\partial}{\partial t}(\tilde{\rho}s)^* - [\rho s]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} =$$

$$= \left[-\rho u s + \frac{\varepsilon}{\sigma L}\frac{\partial s}{\partial x}\right]_{\zeta-0}^{\zeta+0} - \varepsilon \frac{\partial}{\partial y}(\rho v s)^* - \left[-\rho v s + \frac{\varepsilon}{\sigma L}\frac{\partial s}{\partial y}\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} - M \qquad (3.13)$$

Let us linearize the boundary conditions (3.10) - (3.13) for the case of small perturbations

$$\left[p' + \rho' U^2 + RU\left(2u' - \frac{\partial \zeta}{\partial t}\right)\right]_{-0}^{+0} =$$

$$= \varepsilon\left\{\left[\frac{4}{3}\frac{\partial u'}{\partial x} - \frac{2}{3}\frac{\partial v'}{\partial y}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Ru')^* - \frac{\partial}{\partial t}(\rho' U)^* - \frac{\partial}{\partial y}(RUv')^*\right\}$$

$$\left[RUv' - P\frac{\partial \zeta}{\partial y}\right]_{-0}^{+0} = \varepsilon\left\{\left[\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Rv')^* - \frac{\partial}{\partial y}(p')^*\right\}$$

$$\left[\rho' U + R\left(u' - \frac{\partial \zeta}{\partial t}\right)\right]_{-0}^{+0} = \varepsilon\left\{-\frac{\partial}{\partial t}(\rho')^* - \frac{\partial}{\partial y}(Rv')^*\right\}$$

$$\left[RU\theta' + \rho' U\Theta + R\Theta\left(u' - \frac{\partial \zeta}{\partial t}\right)\right]_{-0}^{+0} =$$

$$= \varepsilon\left\{\left[\frac{1}{\sigma}\frac{\partial \theta'}{\partial x}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(R\theta')^* - \frac{\partial}{\partial t}(\rho'\Theta)^* - \frac{\partial}{\partial y}(R\Theta v')^*\right\} + M'$$

$$\left[RUs' + \rho' US + RS\left(u' - \frac{\partial \zeta}{\partial t}\right)\right]_{-0}^{+0} =$$

$$= \varepsilon\left\{\left[\frac{1}{\sigma L}\frac{\partial s'}{\partial x}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Rs')^* - \frac{\partial}{\partial t}(\rho' S)^* - \frac{\partial}{\partial y}(RSv')^*\right\} - M' \qquad (3.14)$$

For furthest calculations of the quantities ( )* it is necessary to make use of the concrete structure of the flame front. We will accept the fact that the structure of the flame has the form given by the theory of Ya. B. Zel'dovich and D. A. Frank-Kamenetskiy [13, 14]. In this theory it is assumed that the activation energy of the chemical reaction of combustion is extremely large; the flame consists of two zones -- a warm-up zone (the thickness of this zone is of the order ε) and a zone of chemical reaction, thin relative to the warm-up zone. Consistent with this, we will neglect the thickness of the reaction zone and consider that the speed of reaction is determined only by the temperature of the products of combustion leaving this zone ($M = M(\theta_r)$). Under these circumstances the temperature θ_r can, generally speaking, differ from the adiabatic temperature of combustion in the stationary case ($\theta_r \neq \Theta = 1$) (see, for example, [15, 16]).

The structure of the flame front is described by a system of hydrodynamic, diffusion and heat transfer equations (2.1). Under these conditions, for the /460 curvature of the flame front it is convenient to take equations in orthogonal curvilinear coordinates, connected directly to the flame front, since within the flame structure the distribution of values in the front is understood to be along the normal to its surface, n. Besides, it is necessary to interchange the coordinates n with the coordinates $\eta = n/\varepsilon$. Since for calculation of the

quantities ( )* it is sufficient to know the flame structure in the zero-th approximation, after interchange of n with η we can then neglect terms of the order ε. As can be readily proved, the time and space derivatives directed along the flame front in the equations will disappear for this reason. The Lame constants will become unity. For small curvatures of the flame front with accuracies of terms of higher orders the normal directions will coincide with the x axis; therefore, we can consider η = n/ε = ξ = (x − ζ)/ε.

Then from equations (2.1) we get (outside the zone of chemical reaction)

$$\frac{\partial}{\partial\xi}\left(-p-\rho v_n^2+\frac{4}{3}\frac{\partial v_n}{\partial\xi}\right)=0,\quad \frac{\partial}{\partial\xi}\left(-\rho v_n v_\tau+\frac{\partial v_\tau}{\partial\xi}\right)=0,\quad \frac{\partial}{\partial\xi}(\rho v_n)=0$$
$$\frac{\partial}{\partial\xi}\left(-\rho v_n\theta+\frac{1}{\sigma}\frac{\partial\theta}{\partial\xi}\right)=0,\quad \frac{\partial}{\partial\xi}\left(-\rho v_n s+\frac{1}{\sigma L}\frac{\partial s}{\partial\xi}\right)=0 \tag{3.15}$$

where ν_n and ν_τ are the additive velocities of the current, normal and tangent to the front, respectively; they are related to the velocities u and ν by the relations

$$v_n=V_n+v_n'=U+u'-\frac{\partial\zeta}{\partial t},\qquad v_\tau=v'+U\frac{\partial\zeta}{\partial y} \tag{3.16}$$

It is necessary to supplement the system (3.15) by the equations of state (2.2).

The boundary conditions for the system of equations (3.15) will be: 1) continuity conditions. When $\xi\to\pm\infty$ the solutions must go over into the values corresponding to the values on the surface of the flame, arrived at by L. D. Landau (formulas (2.6) and (2.8) when x = 0); and 2) conditions on the reaction zones. Here we require continuity of tangential velocity, temperature and concentration (the conservation conditions here will be automatically satisfied). The solution of the system with consideration of linearization and the noted boundary conditions have the form

for the stationary quantities

$$U_-=1+\frac{1-\alpha}{\alpha}e^{\sigma\xi},\quad U_+=\frac{1}{\alpha},\quad V_-=V_+=0$$
$$P_-=P_{-\infty}+\frac{1-\alpha}{\alpha}\left(\frac{4}{3}\sigma-1\right)e^{\sigma\xi},\quad P_+=P_{-\infty}-\frac{1-\alpha}{\alpha}$$
$$\Theta_-=e^{\sigma\xi},\quad \Theta_+=1,\quad S_-=1-e^{\sigma L\xi},\quad S_+=0$$
$$R_-=1\Big/\left(1+\frac{1-\alpha}{\alpha}e^{\sigma\xi}\right),\quad R_+=\alpha \tag{3.17}$$

for the perturbed quantities

$$u_-' = \left\{A^0 + \frac{1-\alpha}{\alpha}\left[(A^0 - \omega D)(1+\sigma\xi) + H^0\right] e^{\sigma\xi}\right\} e^{i\gamma+\omega t}, \quad u_+' = (B^0 + C^0)\, e^{i\gamma+\omega t}$$

$$v_-' = i\left[A^0 - \left(A^0 + B^0 + \alpha\omega C^0 - \frac{1-\alpha}{\alpha} D\right) e^{\xi} - \frac{1-\alpha}{\alpha} D e^{\sigma\xi}\right] e^{i\gamma+\omega t}$$

$$v_+' = -i\,(B^0 + \alpha\omega C^0)\, e^{i\gamma+\omega t}$$

$$p_-' = \left\{-A^0(\omega+1) + \frac{1-\alpha}{\alpha}\left(\frac{4}{3}\sigma - 1\right)\left[(A^0 - \omega D)(2+\sigma\xi) + H^0\right] e^{\sigma\xi}\right\} e^{i\gamma+\omega t} \qquad /461$$

$$p_+' = B^0(\alpha\omega - 1)\, e^{i\gamma+\omega t}, \qquad \rho_+' = -\alpha(1-\alpha) H^0 e^{i\gamma+\omega t}$$

$$\rho_-' = -\frac{1-\alpha}{\alpha}\left[(A^0 - \omega D)\sigma\xi + H^0\right]\left(1 + \frac{1-\alpha}{\alpha} e^{\sigma\xi}\right)^{-2} e^{\sigma\xi} e^{i\gamma+\omega t}$$

$$\theta_-' = \left\{\left[(A^0 - \omega D)\sigma\xi + H^0\right] e^{\sigma\xi}\right\} e^{i\gamma+\omega t}, \quad \theta_+' = H^0 e^{i\gamma+\omega t}$$

$$s_-' = -(A^0 - \omega D)\,\sigma L\xi e^{\sigma L\xi} e^{i\gamma+\omega t}, \quad s_+' = 0 \qquad (3.18)$$

Considering formulas (3.17) and (3.18), we can evaluate the quantities ( )* entering boundary conditions (3.14). We get

$$(p')^* = \frac{1-\alpha}{\alpha}\frac{1}{\sigma}\left(\frac{4}{3}\sigma - 1\right)(A^0 - \omega D + H^0)\, e^{i\gamma+\omega t}$$

$$(\rho')^* = -\frac{1}{\sigma}\left[(A^0 - \omega D)\ln\alpha + (1-\alpha) H^0\right] e^{i\gamma+\omega t}$$

$$(Ru')^* = \frac{1}{\sigma}\left\{A^0 \ln\alpha - \left[I_1(\alpha) + \ln\alpha\right](A^0 - \omega D) - H^0 \ln\alpha\right\} e^{i\gamma+\omega t}$$

$$(Rv')^* = i\left\{\frac{\ln\alpha}{\sigma} A^0 - \frac{\alpha(A^0 + B^0 + \alpha\omega C^0)}{1-\alpha} I_2(\alpha,\sigma) + \left[I_2(\alpha,\sigma) + \frac{\ln\alpha}{\sigma}\right] D\right\} e^{i\gamma+\omega t}$$

$$\left(I_1(\alpha) \equiv \int_0^\infty \frac{te^{-t}\,dt}{\alpha/(1-\alpha) + e^{-t}}, \qquad I_2(\alpha, m) \equiv \int_0^\infty \frac{e^{-t}\,dt}{\alpha/(1-\alpha) + e^{-mt}}\right)$$

$$(R\theta')^* = -\frac{\alpha}{\sigma(1-\alpha)}\left[I_1(\alpha)(A^0 - \omega D) + H^0 \ln\alpha\right] e^{i\gamma+\omega t}$$

$$(Rs')^* = \frac{\alpha}{1-\alpha}\frac{1}{\sigma L} I_3\left(\alpha, \frac{1}{L}\right)(A^0 - \omega D)\, e^{i\gamma+\omega t} \left(I_3(\alpha, m) \equiv \int_0^\infty \frac{te^{-t}\,dt}{\alpha/(1-\alpha) + e^{-mt}}\right)$$

$$(\rho' U)^* = \frac{1}{\sigma}\left[I_1(\alpha)(A^0 - \omega D) + H^0 \ln\alpha\right] e^{i\gamma+\omega t}$$

$$(\rho'\Theta)^* = \frac{1}{\sigma}\left\{\frac{\alpha}{1-\alpha}\left[I_1(\alpha) + \ln\alpha\right](A^0 - \omega D) + \left(\alpha + \frac{\alpha}{1-\alpha}\ln\alpha\right) H^0\right\} e^{i\gamma+\omega t}$$

$$(\rho' S)^* = \frac{1}{\sigma}\left\{\left[-\ln\alpha + \frac{\alpha}{1-\alpha}\frac{1}{L} I_2\left(\alpha, \frac{1}{L}\right) - \frac{\alpha}{1-\alpha}\frac{1}{L} I_3\left(\alpha, \frac{1}{L}\right)\right](A^0 - \omega D) + \left[-1 + \frac{\alpha}{1-\alpha} I_2\left(\alpha, \frac{1}{L}\right)\right] H^0\right\} e^{i\gamma+\omega t}$$

$$(RUv')^* = i\left[-(A^0 + B^0 + \alpha\omega C^0) + \frac{1-\alpha}{\alpha}\left(1 - \frac{1}{\sigma}\right) D\right] e^{i\gamma+\omega t}$$

$$(R\Theta v')^* = i\left\{-\frac{\alpha}{1-\alpha}\frac{\ln\alpha}{\sigma} A^0 - \frac{\alpha}{1-\alpha}\frac{1}{\sigma+1} I_2\left(\alpha, \frac{\sigma}{\sigma+1}\right)(A^0 + B^0 + \alpha\omega C^0) - \left[-\frac{1}{\sigma+1} I_2\left(\alpha, \frac{\sigma}{\sigma+1}\right) + \frac{\alpha}{1-\alpha}\frac{\ln\alpha}{\sigma} + \frac{1}{\sigma}\right] D\right\} e^{i\gamma+\omega t}$$

$$(RSv')^* = i\left\{\frac{\ln\alpha}{\sigma}A^0 - \frac{\alpha}{1-\alpha}\frac{1}{\sigma L}I_2\left(\alpha, \frac{1}{L}\right)A^0 - \frac{\alpha}{1-\alpha}\left[I_2(\alpha, \sigma) - \right.\right.$$
$$\left.\left. - \frac{1}{\sigma L+1}I_2\left(\alpha, \frac{\sigma}{\sigma L+1}\right) + \frac{\ln\alpha}{\sigma} + \frac{1}{\sigma(L+1)}I_2\left(\alpha, \frac{1}{L+1}\right)\right]D\right\}e^{i\omega t + \omega t} \qquad (3.19)$$

The calculations carried out with the quantities ( )* (formulas (3.19)) /462 complete the development of the boundary conditions on the flame surface with an accuracy up to the first approximation in the quantity ε.

§4. Let us now go to the evaluation of the characteristic frequency of the problem. Satisfying the solution of equations (2.6), (2.8) and (2.10) by boundary conditions (3.14) and equating coefficients in the zero-th and first levels of ε, we will get two systems of linear first order algebraic equations. The first system, which results from equating the coefficients to zero in the zero-th level, serves to evaluate the constant entering the zero-th approximation

$$A^0(\omega - 1) + B^0(\alpha\omega + 1) + 2C^0 - \frac{1-\alpha}{\alpha}H^0 = 0$$
$$-A^0 - B^0 - \alpha\omega C^0 + \frac{1-\alpha}{\alpha}D = 0$$
$$-A^0 + \alpha B^0 + \alpha C^0 + (1-\alpha)\omega D - (1-\alpha)H^0 = 0$$
$$\alpha B^0 + \alpha C^0 - \alpha\omega D + \alpha H^0 = 0, \quad -A^0 + \omega D = 0 \qquad (4.1)$$

From the last three equations, as is easily seen, it follows that $H^0 = 0$. In this way, in the solution of the problem in the zero-th approximation there is no perturbation of temperature behind the flame. This is physically understandable, since diffusion and heat conduction, which can bring on temperature perturbation, are ignored.

Omitting the fourth equation (4.1) and assuming $H^0 = 0$, we get a system of equations arrived at by L. D. Landau (in several other combinations of equations). Equating the determinant of the system to zero to get a nontrivial solution, we arrive at the characteristic equation of L. D. Landau

$$(\Omega^0)^2 + \frac{2\alpha}{1-\alpha}\Omega^0 - \frac{\alpha(1-\alpha)}{1+\alpha} = 0 \qquad (\Omega^0 \equiv \alpha\omega^0) \qquad (4.2)$$

Assuming in the system that $\omega = \omega^0$, we find the expression for the coefficients A^0, B^0, C^0 through the amplitude of perturbation of the surface D. We have

$$A^0 = \omega^0 D, \qquad B^0 = \frac{1-\alpha}{\alpha(1+\alpha)}D, \qquad C^0 = \left(\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right)D \qquad (4.3)$$

Let us pass over to the solution of the problem in the first approximation. Equating the coefficients in ε, we will get

$$(\omega^0 - 1) A^1 + (\alpha\omega^0 + 1) B^1 + 2C^1 - \frac{1-\alpha}{\alpha} H^1 = A^0\omega^0\tau + B^0\alpha\omega^0\tau + \{\}_1 \quad (4.4.1)$$

$$-A^1 - B^1 - \alpha\omega^0 C^1 = [1 - \alpha\omega^0\tau - (\alpha\omega^0)^2] C^0 + \{\}_2 \quad (4.4.2)$$

$$-A^1 + \alpha B^1 + \alpha C^1 - (1-\alpha) H^1 = (1-\alpha)\omega^0\tau D + \{\}_3 \quad (4.4.3)$$

$$\alpha B^1 + \alpha C^1 + \alpha H^1 = -\alpha\omega^0\tau D + \{\}_4 \quad (4.4.4)$$

$$-A^1 = \omega^0\tau D + \{\}_5 \quad (4.4.5)$$

In writing out this system a new notation was utilized: the right side of the boundary conditions (3.14) for brevity were designated $\varepsilon\{\}_k$, where the lower index indetifies the equation number. Besides this, the presentation of the characteristic frequency of the problem in form (1.3) was used (the first approximation in ε is sought also for the characteristic frequency in the problem). /463

Let us note that from the system (4.4) it is easy to find H^1. Combining (4.4.3), (4.4.4) and (4.4.5) we get

$$H^1 = -\{\}_3 + \{\}_4 + \{\}_5 \quad (4.5)$$

To evaluate the right sides of the system (4.4) through the magnitude of the amplitude of perturbation of the flame surface we will use the solution of the zero-th approximation, specifically, relation (4.3). We get for the quantities $(\ )^*$

$$(p')^* = (\rho')^* = (R\theta')^* = (Rs')^* = (\rho' U)^* = (\rho'\Theta)^* = (\rho' S)^* = 0$$

$$(Ru')^* = \omega^0 \frac{\ln\alpha}{\sigma} D e^{iy+\omega t}, \quad (Rv')^* = i(\omega^0 + 1)\frac{\ln\alpha}{\sigma} D e^{iy+\omega t}$$

$$(RUv')^* = -i\frac{1-\alpha}{\alpha}\frac{1}{\sigma} D e^{iy+\omega t}$$

$$(RSv')^* = \frac{i}{\sigma}\left[(\omega^0 + 1)\ln\alpha - \frac{\alpha}{1-\alpha}\frac{\omega^0}{L} I_2\left(\alpha, \frac{1}{L}\right) + \frac{1}{L+1} I_2\left(\alpha, \frac{1}{L+1}\right)\right] D e^{iy+\omega t}$$

$$(R\Theta v')^* = -\frac{i}{\sigma}\left[1 + \frac{\alpha(\omega^0 + 1)}{1-\alpha}\ln\alpha\right] D e^{iy+\omega t} \quad (4.6)$$

and with the use of (4.6) for the right hand sides of system (4.4)

$$\{\}_1 = -\left[\left(2 + \frac{1}{\sigma}\right)\frac{1-\alpha}{\alpha} + \frac{(\omega^0)^2}{\sigma}\ln\alpha\right] D$$

$$\{\ \}_2 + [1-(\alpha\omega^0)^2]\,C^0 = -\frac{\omega^0(\omega^0+1)}{\sigma} D\ln\alpha, \qquad \{\ \}_3 = \frac{\omega^0+1}{\sigma} D\ln\alpha$$

$$\{\ \}_4 = -\frac{1}{\sigma}\left[1+\frac{\alpha}{1-\alpha}(\omega^0+1)\ln\alpha\right]D + zH^1$$

$$\{\ \}_5 = \frac{1}{\sigma}\left[(\omega^0+1)\ln\alpha - \frac{\alpha}{1-\alpha}\frac{\omega^0}{L} I_2\left(\alpha, \frac{1}{L}\right) + \frac{1}{L+1} I_2\left(\alpha, \frac{1}{L+1}\right)\right]D - zH^1 \qquad (4.7)$$

while for H^1 the relation

$$H^1 = -\frac{1}{\sigma}\left\{[\ ]_a + \frac{\alpha\omega^0}{\alpha-1}[\ ]_b\right\}D = -KD \qquad (4.8)$$

where for brevity we introduce the designation

$$[\ ]_a \equiv \left[1+\frac{\alpha}{1-\alpha}\ln\alpha - \frac{1}{L-1} I_2\left(\alpha, \frac{1}{L+1}\right)\right], \quad [\ ]_b \equiv \left[\frac{1}{L} I_2\left(\alpha, \frac{1}{L}\right) + \ln\alpha\right] \qquad (4.9)$$

and utilize the relation for speed of reaction from the temperature

$$M' = z\theta_r' = \varepsilon z H^1 \exp(iy + \omega t)$$

(Brackets $[\ ]_{a,b}$ become zero when the Lewis number L = 1).

We combine the equations in such manner that the system will assume the form of the analogous system of L. D. Landau [1, 2]. In this it is necessary /464 to perform the operations

$$-\frac{2}{\alpha}(4.4.1) + 2\frac{1-\alpha}{\alpha}(4.4.5), \quad -(4.4.2), \quad \frac{1}{\alpha}(4.4.3) - \frac{1}{\alpha}(4.4.5), \quad -(4.4.5)$$

The numbers in parentheses correspond to the equations of system (4.4); equation (4.4.4) is omitted -- it was used for determination of H^1. We get

$$(\omega^0+1)A^1 + (z\omega^0-1)B^1 + \left\{\tau\left[\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right] + \frac{1-\alpha}{\alpha}\left(2-\frac{1}{\sigma}\right) + \right.$$
$$\left. + \frac{(\omega^0)^2}{\sigma}\ln\alpha - \frac{1-\alpha}{\alpha}(1-2z)K\right\}D = 0$$
$$A^1 + B^1 + \alpha\omega^0 C^1 - \left\{\alpha\omega^0\tau\left[\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right] + \frac{\omega^0(\omega^0+1)}{\sigma}\ln\alpha\right\}D = 0$$
$$B^1 + C^1 + \left\{\omega^0\tau + \frac{1}{\alpha\sigma} + \frac{\omega^0+1}{(1-\alpha)\sigma}\ln\alpha + \left(1-\frac{z}{\alpha}\right)K\right\}D = 0$$
$$A^1 + \left\{\omega^0\tau + \frac{1}{\sigma}\left[1+\frac{\omega^0+1}{1-\alpha}\ln\alpha\right] + (1-z)K\right\}D = 0 \qquad (4.10)$$

Similarly, as in the solution of the problem in the zero-th approximation, for existance of a nontrivial solution it is necessary that the determinant of the system (4.10) equals zero. This gives the conditions for determination of the correction to the characteristic frequency of the problem τ.

We find

$$\tau = \frac{1-\alpha}{1-\alpha-\Omega^0} - \frac{1+\alpha+2\Omega^0}{1-\alpha-\Omega^0}\frac{1}{\sigma}\left(1+\frac{\ln\alpha}{1-\alpha}\right) + \\ + \frac{1+\Omega^0}{1-\alpha-\Omega^0}(1+\alpha-2z)\frac{1}{\sigma}\left\{[\quad]_a + \frac{\Omega^0}{1-\alpha}[\quad]_b\right\} \tag{4.11}$$

If we extrapolate (4.11) in the region of perturbing wavelengths, for which $\omega = 0$, then the quantity τ becomes equal to the critical Reynolds number, constructed along the length of the perturbation wave (more accurately, along wave number $k = 2\pi/\lambda$) $\tau = N_{R_0}$.

§5. The derived formula (4.11) in the previous paragraph shows what influence the separate dissipative effects have on the stabilization of the flame.

The first item in this formula characterizes the influence of viscosity.

As is readily verified, using the expression for Ω^0

$$\Omega^0 = (-\alpha + \sqrt{\alpha+\alpha^2-\alpha^3})/(1+\alpha) \tag{5.1}$$

(see (4.2)), this item always exceeds zero -- viscosity in this way always stabilizes the flame. This agrees with the derivations of works [5, 6] and contradicts the results of works [4, 8], in which there was developed a physically strange effect of a destabilizing influence of viscosity. The critical Reynolds number, derived from the effects of viscosity

$$N_{R1} = (1-\alpha)/(1-\alpha-\Omega^0) \tag{5.2}$$

is not large. Figure 2 shows the dependence of N_{R_1} on α. For real flames, $\alpha = 0.1$ to 0.2 and $N_{R_1} = 1.3$ to 1.5.

The second item in formula (4.11) characterizes the influence of heat con-/465 duction. The critical Peclet number, depending solely on this influence, equals

$$\Pi_2 = \sigma N_{R2} = -\frac{1+\alpha+2\Omega^0}{1-\alpha-\Omega^0}\left(1+\frac{\ln\alpha}{1-\alpha}\right) \tag{5.3}$$

The dependence of Π_2 on α is similarly represented in Figure 2. Heat transfer similarly only stabilizes the flame. For real flames the value of the Peclet numbers $\Pi_2 \approx 3$.

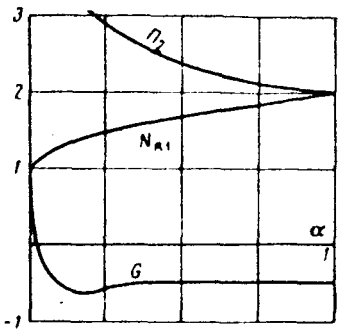

Figure 2.

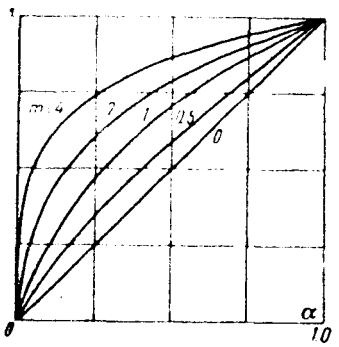

Figure 3.

Finally, the last item in formula (4.11) describes the influence on flame stability of the relation between diffusion and heat transfer. The critical Peclet number, describing this influence, equals

$$\Pi_3 = \frac{1+\Omega^0}{1-\alpha-\Omega^0}(1+\alpha-2z)\left\{[\quad]_a + \frac{\Omega^0}{1-\alpha}[\quad]_b\right\} \qquad (5.4)$$

It depends on two parameters -- the degree of heat divergence and the Lewis numbers L, and characterizes in this manner the influence of the relationship between diffusion and heat conduction.

It is necessary to stress,nevertheless, that the role of the Lewis number here is not at all different than in the case of completely diffusional thermal stability [15, 16]. The latter is connected with the predetermination of heat and mass between the bulges and depressions of the front resulting from the transverse diffusional and conductive thermal flows, arising from the curvature of the flame. In our case the transverse thermal currents in the substance are convective: the flame front is examined here in approaching the boundary layer, the diffusion and heat conductivity tend only in a direction normal to the front (as quasi-stationary changes of concentration and temperature profiles), while along the length of the front, heat and mass are transferred by pulsations of tangential velocity. In this way, diffusional-thermal phenomena in the sense of works [15, 16] will be manifested only during evaluation of corrections of the second order in ε to the solution of L. D. Landau. The role of the Lewis numbers in the given problem impinge on the fact that the diffusional and thermal width of the front are different and because of this the effective values of the concentration and temperature are different, these being convergent due to the transverse pulses. In Figure 1 it is seen how the inside of the flame bends the current lines, and the comprising velocities along the y axis change signs. Depending on the level of thermal divergence α, the transfer

on the internal or external part of the front can govern, and the effect will differ during identical relationships between diffusion and thermal conductivity.

This can be easily demonstrated. If $\alpha \to 0$ (strong thermal divergence), /466 then for the Peclet number we will get

$$\Pi_3 \to (1-2z)(L-1)/L \approx -2z(L-1)/L \tag{5.5}$$

When $\alpha \to 1$ (no thermal divergence)

$$\Pi_3 \to -2(1-z)(L-1)/L(L+1) \approx 2z(L-1)/L(L+1) \tag{5.6}$$

and with the selfsame value of L-1 for the Peclet number the sign changes.

If the Lewis number is close to unity $|L-1| \ll 1$, in place of formula (5.4) we can get a much simpler relationship

$$\Pi_3 \approx -2z\frac{1+\Omega^0}{1-\alpha-\Omega^0}\left[1-\frac{\Omega^0+\alpha}{1-\alpha}I_1(\alpha)\right](L-1) = -2zG(\alpha)(L-1) \tag{5.7}$$

The function $G(\alpha)$ is pictured in Figure 2. It is evident that it goes through zero with very small α's, which are not encountered in practice.

In the general case, it is necessary to utilize formula (5.4). To simplify the calculations with this formula, Figure 3 shows the plotted curves $[\alpha/(1-\alpha)]\, I_2(\alpha, m)$ for different values of parameter m.

In this manner, consideration of the effects of transfer leads to stabilization of small perturbations. The critical Reynolds numbers τ, developed through extrapolation of linear corrections, do not differ markedly from unity. For example, when $\alpha = 0.2$, $z = 10$, $\sigma = 1$ and $L = 1.2$, we get $\tau \approx 7$.

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